Quantum Black Hole

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Abstract

Creation of a black hole in quantum cosmology is the third way of black hole formation. In contrast to the gravitational collapse from a massive body in astrophysics or from the quantum fluctuation of matter fields in the very early universe, in the quantum cosmology scenario the black hole is essentially created from nothing. The black hole originates from a constrained gravitational instanton. The probability of creation for all kinds of single black holes in the Kerr-Newman family, at the semi-classical level, is the exponential of the total entropy of the universe, or one quarter of the sum of both the black hole and the cosmological horizon areas. The de Sitter spacetime is the most probable evolution at the Planckian era.

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It is well known in astrophysics that a black hole can be formed in two ways. The first is the gravitational collapse of a massive body. If the mass of a star exceeds about twice that of the Sun, a black hole will be its ultimate corpse. The second way of formation of a so-called primordial black hole originates from the fluctuation of matter distribution in the early universe. In the Big Bang model, the matter content can be classically described [1][2], while in the inflationary universe the matter content is attributed to the quantum fluctuation of the Higgs scalar [3]. The mass for the black hole formed in this way is very low.

Strictly speaking, black holes formed through the second way can hardly be regarded as primordial. A true primordial black hole should be created at the moment of the birth of the universe. Therefore, we are introducing the third way, i.e. the black hole creation in the quantum cosmology scenario. In this scenario both spacetime and matter fields are quantized, and most dramatically, the black hole is essentially created from nothing.

Over the last decade there have been several attempts to deal with this problem; however, their results are not conclusive [4][5]. Recently, many studies have been carried out for the creation of black hole pairs [6][7][8][9][10][11]. However, the most interesting case is the creation of a single primordial black hole, which is the topic of this article.

In the No-Boundary Universe a Lorentzian evolution of the universe emanates from Euclidean manifolds through a quantum transition at a 3-surface Σ with the matter field ϕ on it. Its probability can be written as a path integral [12]

$$P = \Psi^\star \Psi = \int_C d[g_{\mu\nu}] d[\phi] \exp(-\bar{I}([g_{\mu\nu}, \phi]), \tag{1}$$

where class C is all no-boundary compact Euclidean 4-metrics and matter field configurations which agree with the given 3-metric h_{ij} and matter field ϕ on Σ. Here $\bar{I}$ means the Euclidean action.

The Euclidean action for the gravitational part for a smooth spacetime manifold M with boundary ∂M is

$$\bar{I} = -\frac{1}{16\pi} \int_M (R - 2\Lambda) - \frac{1}{8\pi} \int_{\partial M} K, \tag{2}$$

where Λ is the cosmological constant, R is the scalar curvature, K is the trace of the second fundamental form of the boundary.

Here, we do not restrict class C to contain regular metrics only, since the derivation of Eq. (1)

from the ground state proposal of Hartle and Hawking has already inevitably led to some jump discontinuities in the extrinsic curvature at Σ. To make the theory consistent, one has to allow the discontinuity to occur anywhere in M.

The dominant contribution to the path integral (2) comes from stationary action trajectories, which are the saddle points of the path integral. The stationary action trajectories should meet all requirements on the 3-surface Σ and other restrictions. At the WKB level, the exponential of the negative of the stationary action is the probability of the corresponding Lorentzian trajectory.

In some sense, the set of all regular metrics is not complete, since for many cases, under the usual regularity conditions and the requirements at the equator Σ, there may not exist any stationary action metric, i.e. a gravitational instanton which is defined as a Euclidean solution to the Einstein field equation. It seems reasonable to include metrics with jump discontinuities of extrinsic curvature and with their degenerate cases, i.e. the conical singularities, into class C [13]. Within the extended class C one can hopefully find a stationary action trajectory, i.e. a constrained gravitational instanton with some mild singularities in the absence of a regular instanton. For our consideration, the singularity can only occur at some locations on the given 3-metric Σ. The stationary action trajectory satisfies the usual Einstein field equation except for the singularities. One can rephrase this by saying that the metric obeys the generalized Einstein equation in the whole manifold. The extrinsic curvature will not vanish at the singularity locations at Σ. Since this result is derived from first principles and one is dealing with the action itself, instead of the Einstein equation, in quantum cosmology, one should not feel upset about this situation.

In general, a wave packet of the wave function of the universe represents an ensemble of classical trajectories. Under our scheme, the most probable trajectory associated with an instanton can be singled out [14]. Thus, quantum cosmology obtains its complete power of prediction. It means there is no further degree of freedom except for a physical time as long as the model is well-defined.

It is believed that the Planckian era of the universe can be described by a de Sitter spacetime with some effective cosmological constant Λ [15]. Therefore, one is interested in the black hole creation in this background. If there is no black hole in the universe, then one can get a regular instanton S^4. If there is, then the restrictions are strong enough to allow one to have a constrained nonregular instanton only, and the corresponding stationary action will take a relatively greater

value. Therefore, the probability of a universe with a black hole is always smaller than one without a black hole.

First, we can consider the spherically symmetric vacuum case [16]. The Euclidean Schwarzschild-de Sitter metric with mass parameter m is [17],

$$ds^2 = \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right) d\tau^2 + \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{3}$$

In general, one can use r_2, r_3 to denote the black hole and cosmological horizons, which are the two positive roots of the expression $1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}$ for this case. If $0 \leq m \leq m_c = \Lambda^{-1/2}/3$, then one has an Euclidean sector $r_2 \leq r \leq r_3$. For the extreme case $m = m_c$ the sector degenerates into the $S^2 \times S^2$ Nariai space. The Nariai spacetime is identified as a pair of black holes punched through the S^4 space.

In the $(\tau - r)$ plane $r = r_2$ is an axis of symmetry and the imaginary time coordinate τ is identified with period β_2, whose reciprocal is the Hawking temperature. This makes the Euclidean manifold regular at the black hole horizon. One can also apply this procedure to the cosmological horizon with period β_3, whose reciprocal is the Gibbons-Hawking temperature. For the $S^2 \times S^2$ case these two horizons are identical, thus one obtains a regular instanton. Except for the $S^2 \times S^2$ spacetime, one cannot simultaneously regularize it at both horizons because of the inequality $\beta_2^{-1} > \beta_3^{-1}$.

Now we are going to construct a constrained gravitational instanton. One can have two cuts at $\tau = consts.$ between $r = r_2$ and $r = r_3$. Then the f_2-fold cover turns the $(\tau - r)$ plane into a cone with a deficit angle $2\pi(1 - f_2)$ at the black hole horizon. In a similar way one can have an f_3-fold cover at the cosmological horizon. Both f_2 and f_3 can take any pair of real numbers with the relation

$$f_2\beta_2 = f_3\beta_3. \tag{4}$$

This manifold satisfies the usual Einstein equation except for the conical singularities at at least one of the two horizons. The variation calculation of the action requires that the manifold obeys the Einstein equation everywhere with the possible exception at the transition surface where the constraints are imposed. We assume the quantum tunneling will occur at the equator which are two $\tau = const.$ sections, say $\tau = \pm f_2\beta_2/4$, passing through the two horizon. Therefore to check whether we have obtained a constrained instanton, it is only necessary to prove that the action is stationary

with respect to the parameter f_2 or f_3, i.e, the only degree of freedom left.

If f_2 or f_3 is different from 1, then the cone at the black hole or cosmological horizon will have an extra contribution to the action of the manifold. Since the integral of K with respect to the 3-area in the boundary term of the action (2) is the area increase rate along its normal, then the extra contribution due to the conical singularities can be considered as the degenerate form shown below

$$\bar{I}_{i,cone} = -\frac{1}{8\pi} \cdot 4\pi r_i^2 \cdot 2\pi(1 - f_i). \quad (i = 2, 3) \tag{5}$$

Thus, the total action can be calculated

$$\bar{I}_{total} = -\frac{f_2 \beta_2 \Lambda}{6}(r_3^3 - r_2^3) + \sum_{i=2,3} \bar{I}_{i,cone}, \tag{6}$$

where the first term of the right hand side is due to the volume contribution.

Substituting Eqs. (4) and (5) into Eq. (6), one can obtain

$$\bar{I}_{total} = -\pi(r_2^2 + r_3^2). \tag{7}$$

This is one quarter of the negative of the sum of these two horizon areas. One quarter of the sum is the total entropy of the universe.

It is remarkable to note that the action is independent of the choice of f_2 or f_3. This means that our constructed manifold has a stationary action and is qualified as a constrained gravitational instanton. Therefore it can be used for the WKB approximation to the wave function. This phenomenon also occurs for the whole family of Kerr-Newman black holes as we shall discuss below. Nature has a great propensity for black holes! Therefore, the creation probability of a Schwarzschild black hole in the de Sitter background, at the WKB level, is

$$P_m \approx \exp(\pi(r_2^2 + r_3^2)). \tag{8}$$

Our result implies that no matter which flat fragment of the manifold is chosen, the same black hole should be created with the same probability. Of course, the most dramatic case is that of no volume, i.e. $f_2 = f_3 = 0$.

Formula (8) interposes the following two extreme cases [9]. First for the de Sitter case with $m = 0$, $P_0 \approx \exp(3\pi\Lambda^{-1})$, and second for the Nariai case with $m = m_c$, $P_{m_c} \approx \exp(2\pi\Lambda^{-1})$.

The Schwarzschild black hole case is the simplest, since one can easily glue the north portion and the south portion of the instanton at the equator Σ. One can also consider the Reissner-Nordström black hole case[16]. When the black hole is magnetically charged, then the matter field is represented by the vector potential $A = Q(1 - \cos\theta)d\phi$ over the S^2 space, $(\phi - \theta)$ sector, where Q is the charge. There is no obstacle to gluing, since the magnetic field is continuous at Σ. For the electrically charged case, one can choose the vector potential $A = -iQr^{-2}\tau dr$. Since the vector potential A at Σ does not uniquely define the electric field there, then the formula (1) does not represent the probability of a black hole with an electric charge. In fact, the configuration of the wave function is the three geometry and the momentum ω, which is canonically conjugate to the electric charge Q and is defined by the integral of A around S^1 sector of Σ. Then one can get the wave function $\Psi(Q, h_{ij})$ for the given electric charge through the Fourier transformation [10][11]

$$\Psi(Q, h_{ij}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega e^{i\omega Q}\Psi(\omega, h_{ij}). \tag{9}$$

At the WKB level, the transformation is equivalent to adding the following extra term

$$f_2\beta_2 Q^2(r_2^{-1} - r_3^{-1}) \tag{10}$$

to the action

$$\bar{I} = -\frac{f_2\beta_2\Lambda}{6}(r_3^3 - r_2^3) \pm \frac{f_2\beta_2 Q^2}{2}(r_2^{-1} - r_3^{-1}) + \sum_{i=2,3}\bar{I}_{i,cone}, \tag{11}$$

where $+$ is for the magnetic case and $-$ is for the electric case. Therefore, the Fourier transformation will iron out the sign difference of the action terms due to magnetic and electric fields and thus recover the duality between magnetically and electrically charged black holes. The total action is stationary again. The probability for Reissner-Nordström black hole creation is also expressed by formula (8). All known results on the probability of black hole creation [6][7][8][9][10][11] can be derived as special cases from this quite universal formula.

A similar consideration should be made for the rotation of a black hole [16]. Again, the 3-geometry of Σ determines the angular differentiation between the black hole and cosmological horizons instead of the angular momentum of the hole. One has to use another Fourier transformation relating them to obtain the wave function for a given angular momentum. The probability of Kerr-Newman black hole creation is also the exponential of one quarter of the sum of the black hole and cosmological horizon areas, or the exponential of the total entropy of the universe.

Our calculation has very clearly shown that the gravitational entropy is associated with the spacetime topology. From the no-hair theorem, a stationary black hole in the de Sitter spacetime background is characterized by three parameters only, mass, charge and angular momentum, so the problem of quantum creation of a single black hole at the birth of the universe is completely resolved.

It can be shown that the probability is an exponentially decreasing function of the mass, magnitude of charge and angular momentum of the black hole. Therefore, the de Sitter spacetime is the most probable evolution of the universe at the Planckian era.

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